

# AN ALGEBRAIC MODEL OF BARYON SPECTROSCOPY

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We discuss recent calculations of the mass spectrum, electromagnetic and strong couplings of baryon resonances. The calculations are done in a collective constituent model for the nucleon, in which the resonances are interpreted as rotations and vibrations of a symmetric top with a prescribed distribution of the charge and magnetization. We analyze recent data on eta-photo- and eta-electroproduction, and the tensor analyzing power in deuteron scattering.

## 1 Introduction

Effective models of baryons based on three constituents share a common spin-flavor-color structure but differ in their treatment of the spatial dynamics. Quark potential models in nonrelativistic<sup>1</sup> or relativized<sup>2</sup> forms emphasize the single-particle aspects of quark dynamics for which only a few low-lying configurations in the confining potential contribute significantly to the eigenstates of the Hamiltonian. On the other hand, some regularities in the observed spectra (*e.g.* linear Regge trajectories, parity doubling) hint that an alternative, collective type of dynamics may play a role in the structure of baryons.

In this contribution we present a collective model of baryons within the context of an algebraic approach<sup>3</sup> in which the baryon resonances are interpreted as rotations and vibrations of a symmetric top. We analyze the mass spectrum, the electromagnetic and strong couplings of nonstrange baryons. Particular attention is paid to the new values of the helicity amplitudes of the N(1520) $D_{13}$  and N(1535) $S_{11}$  resonances and their model independent ratios<sup>4,5,6,7,8</sup>, the tensor analyzing power in deuteron scattering<sup>9</sup>, and the strong decays of the N(1535) resonance.

## 2 Algebraic model

We consider baryons to be built of three constituent parts which are characterized by both internal and radial (or spatial) degrees of freedom. The internal degrees of freedom of these three parts are taken to be: flavor-triplet  $u, d, s$  (for the light quark flavors), spin-doublet  $S = 1/2$ , and color-triplet. The internal algebraic structure of the constituent parts consists of the usual spin-flavor (sf) and color (c) algebras  $SU_{sf}(6) \otimes SU_c(3)$ . The difference between models

of baryon spectroscopy lies in the treatment of the spatial dynamics. The relevant radial degrees of freedom for the relative motion of the three constituent parts of this configuration can be taken as the Jacobi coordinates. These can be treated algebraically in terms of the spectrum generating algebra of  $U(7)$  for the radial (or orbital) excitations<sup>3</sup>. The model space is spanned by the symmetric irreducible representation  $[N]$  of  $U(7)$ , and contains the harmonic oscillator shells with  $n = n_\rho + n_\lambda = 0, 1, 2, \dots, N$ . The value of  $N$  determines the size of the model space and, in view of confinement, is expected to be large. The full algebraic structure is obtained by combining the radial part with the internal spin-flavor-color part

$$\mathcal{G} = U(7) \otimes SU_{\text{sf}}(6) \otimes SU_c(3) . \quad (1)$$

The radial part of the baryon wave function has to be combined with the spin-flavor and color parts, in such a way that the total wave function is antisymmetric.

A convenient set of basis states for the baryon wave functions is provided by the case of three identical constituents, for which the radial and spin-flavor parts of the baryon wave function are in addition labeled by their transformation properties under the permutation group  $S_3$ :  $t = S$  for the symmetric,  $t = A$  for the antisymmetric and  $t = M$  mixed symmetry representation. The wave functions can now be labeled as

$$|^{2S+1}\dim\{SU_{\text{f}}(3)\}_J [\dim\{SU_{\text{sf}}(6)\}, L^P], (v_1, v_2); K \rangle . \quad (2)$$

The spin-flavor part is characterized by the dimensions of the irreducible representations of  $SU_{\text{sf}}(6) \supset SU_{\text{f}}(3) \otimes SU_{\text{s}}(2)$ . For the radial part we consider a collective (string-like) model in which the baryons are interpreted as excitations of an oblate top<sup>3</sup>. In this case the spatial part is characterized by the labels:  $(v_1, v_2); K, L_t^P$ , where  $(v_1, v_2)$  denotes the vibrations (stretching and bending) of the oblate top;  $K$  denotes the projection of the rotational angular momentum  $L$  on the body-fixed symmetry-axis;  $P$  the parity and  $t$  the  $S_3$  symmetry type of the state under permutations. The permutation symmetry of the spatial part must be the same as that of the spin-flavor part to ensure the antisymmetry of the total baryon wave-function:  $t = S \leftrightarrow [56]$ ,  $t = A \leftrightarrow [20]$  and  $t = M \leftrightarrow [70]$ . The spin  $S$  and the orbital angular momentum  $L$  are coupled to total angular momentum  $J$ .

### 3 Mass spectrum

The mass operator depends both on the radial and the internal degrees of freedom. The masses of nonstrange baryons have been analyzed with the mass

formula<sup>3</sup>

$$M^2 = M_0^2 + \kappa_1 v_1 + \kappa_2 v_2 + \alpha L + a \left[ \langle \hat{C}_2(SU_{sf}(6)) \rangle - 45 \right] \\ + b \left[ \langle \hat{C}_2(SU_f(3)) \rangle - 9 \right] + c \left[ S(S+1) - \frac{3}{4} \right]. \quad (3)$$

For the radial part we have adopted a collective model of the nucleon in which the baryons are interpreted as rotational and vibrational excitations of an oblate symmetric top. The spectrum consists of a series of vibrational excitations labeled by  $(v_1, v_2)$ , and a tower of rotational excitations built on top of each vibration. The occurrence of linear Regge trajectories suggests to add, in addition to the vibrational frequencies  $\kappa_1$  and  $\kappa_2$ , a term linear in  $L$ . The slope of these trajectories is given by  $\alpha$ . For the spin-flavor part of the mass operator we use a Gürsey-Radicati<sup>10</sup> form. Interaction terms that mix the space and internal degrees of freedom such as for example spin-orbit and tensor couplings have not been included.

The coefficients are determined in a simultaneous fit to all well-established (three and four star<sup>11</sup>) nucleon and  $\Delta$  resonances. The  $N(1440)P_{11}$  and  $\Delta(1600)P_{33}$  resonances are assigned to the  $(v_1, v_2) = (1, 0)$  vibration and the  $N(1710)P_{11}$  resonance to the  $(v_1, v_2) = (0, 1)$  vibration, whereas the negative parity resonances  $N(1520)D_{13}$ ,  $N(1535)S_{11}$  and  $N(1650)S_{11}$  resonances are interpreted as rotational excitations. We find a good overall fit for 24 resonances with a r.m.s. deviation of  $\delta_{\text{rms}} = 39$  MeV<sup>3</sup>.

A common feature to all  $q^3$  quark models is the occurrence of missing resonances. In a recent three-channel analysis evidence was found for the existence of a  $P_{11}$  nucleon resonance at  $1740 \pm 11$  MeV<sup>12</sup>. It is tempting to assign this resonance as one of the missing resonances<sup>13</sup>. In the present calculation it is associated with the  $^28[20, 1^+]$  configuration and appears at 1720 MeV, compared to 1880 MeV in the relativized quark model<sup>2</sup>.

#### 4 Collective form factors

In addition to the masses, it is important to study decay processes which are far more sensitive to details in the baryon wave functions. Under the assumption that the electromagnetic (strong) decays involve the coupling of a photon (elementary meson) to a single constituent, the transition operators that induce these decay processes can be expressed in terms of the algebraic operators<sup>3</sup>

$$\hat{U} = e^{ik\beta \hat{D}_{\lambda,z}/X_D}, \\ \hat{T}_m = -\frac{im_3 k_0 \beta}{2X_D} \left( \hat{D}_{\lambda,m} e^{ik\beta \hat{D}_{\lambda,z}/X_D} + e^{ik\beta \hat{D}_{\lambda,z}/X_D} \hat{D}_{\lambda,m} \right). \quad (4)$$

Here  $(k_0, \vec{k})$  is the four-momentum of the emitted quantum. The dipole operator  $\hat{D}_{\lambda,m}$  is a generator of  $U(7)$  and  $X_D$  its normalization;  $\beta$  is a radial coordinate<sup>3,14</sup>. Different types of collective models are specified by a distribution of the charge and magnetization along the string  $g(\beta)$ . The collective form factors are then obtained by folding the matrix elements of  $\hat{U}$  and  $\hat{T}_m$  with this probability distribution

$$\begin{aligned}\mathcal{F}(k) &= \int d\beta g(\beta) \langle \psi_f | \hat{U} | \psi_i \rangle , \\ \mathcal{G}_m(k) &= \int d\beta g(\beta) \langle \psi_f | \hat{T}_m | \psi_i \rangle .\end{aligned}\tag{5}$$

Here  $\psi$  denotes the spatial part of the baryon wave function. We use the ansatz

$$g(\beta) = \beta^2 e^{-\beta/a} / 2a^3 ,\tag{6}$$

to obtain the dipole form for the elastic form factor. The coefficient  $a$  is a scale parameter. With the same distribution we can now derive closed expressions for the inelastic or transition form factors<sup>3,14</sup>. These collective form factors drop as powers of  $k$ . This property is well-known experimentally and is in contrast with harmonic oscillator quark models in which all form factors which fall off exponentially.

All helicity amplitudes, form factors and decay widths for electromagnetic (transverse, longitudinal and scalar) and strong couplings can be expressed in terms of the radial matrix elements  $\mathcal{F}$  and  $\mathcal{G}_m$ , a spin-flavor matrix element and a phase space factor.

## 5 Electromagnetic couplings

In constituent models, electromagnetic couplings arise from the coupling of the constituent parts to the electromagnetic field. We discuss here the case of the emission of a lefthanded photon from a single constituent

$$B \rightarrow B' + \gamma ,\tag{7}$$

for which the nonrelativistic part of the electromagnetic coupling is given by

$$\mathcal{H}_{em} = 6\sqrt{\frac{\pi}{k_0}} \mu_3 e_3 \left[ k s_{3,-} \hat{U} - \frac{1}{g_3} \hat{T}_- \right] ,\tag{8}$$

where  $e_3$ ,  $\mu_3$ ,  $g_3$  and  $s_3$  are the charge, scale magnetic moment,  $g$  factor and spin of the third constituent. In order to avoid ambiguities arising from the

Table 1: Ratios of helicity amplitudes

|   | N(1535) $S_{11}$<br>$A_{1/2}^n/A_{1/2}^p$ | N(1520) $D_{13}$<br>$A_{3/2}^p/A_{1/2}^p$ |
|---|---|---|
| Feynman et al. <sup>17</sup>  | -0.69                                     | -3.21                                     |
| Koniuk and Isgur <sup>18</sup>                                      | -0.81                                     | -5.57                                     |
| Warns et al. <sup>19</sup>  | -1.06                                     | -9.00                                     |
| Close and Li <sup>20</sup>  | -0.74                                     | -6.55                                     |
|   | -0.65                                     | -4.87                                     |
| Li and Close <sup>21</sup>  | -0.54                                     | -2.50                                     |
|   | -0.56                                     | -2.61                                     |
| Capstick <sup>22</sup>  | -0.83                                     | -8.93                                     |
| Bijker et al. <sup>3</sup>  | -0.69                                     | -2.53                                     |
|   | -0.81 <sup>a</sup>                        |   |
| Santopinto et al. <sup>23</sup>                                     | -0.68                                     | -1.55                                     |
|   | -0.67                                     | -1.80                                     |
| Mukhopadhyay et al. <sup>5,6</sup>                                  | $-0.84 \pm 0.15$                          | $-2.5 \pm 0.2 \pm 0.4$                    |
| Tiator et al. <sup>7</sup>  |   | $-2.1 \pm 0.2$                            |
| PDG <sup>11</sup>   | $-0.51 \pm 0.34$                          | $-6.9 \pm 2.6$                            |
| <sup>a</sup> Calculated with a mixing angle of $\theta = -38^\circ$ |   |   |

treatment of recoil effects, calculations are carried out in the equal momentum or Breit frame. The scale parameter  $a$  of Eq. (6) was determined from a simultaneous fit to the proton and neutron charge radii, and the electric and magnetic form factors of the proton and neutron to be  $a = 0.232$  fm <sup>14</sup>.

### 5.1 Ratios of photocouplings

Recent experiments on eta-photoproduction have yielded valuable new information on the helicity amplitudes of the N(1520) $D_{13}$  and N(1535) $S_{11}$  resonances. In an Effective Lagrangian Approach model-independent ratios of photocouplings were extracted from the new data <sup>5,6</sup>

$$\begin{aligned}
 \text{N(1535)}S_{11} : \quad & A_{1/2}^n/A_{1/2}^p = -0.84 \pm 0.15 \\
 \text{N(1520)}D_{13} : \quad & A_{3/2}^p/A_{1/2}^p = -2.5 \pm 0.2 \pm 0.4
 \end{aligned} \tag{9}$$

The latter value was confirmed in a recent analysis by Tiator et al.<sup>7</sup> who found  $-2.1 \pm 0.2$ . These values are in good agreement with those of the collective algebraic model,  $-0.69$  and  $-2.53$ , respectively<sup>3</sup>.

In the analysis of the photocouplings of the  $N(1535)S_{11}$  and  $N(1650)S_{11}$  resonances it was found that these resonances are mixtures of the  $|^2 8_{1/2}[70, 1^-]\rangle$  and  $|^4 8_{1/2}[70, 1^-]\rangle$  configurations<sup>15</sup>. The transition to the second configuration is forbidden by the Moorhouse selection rule<sup>16</sup>. This mixing breaks  $SU_{sf}(6)$  and can only be introduced by a tensor-like coupling. It appears to be the only place in the spectrum and transitions where there is a clear evidence of such a breaking. With a mixing angle of  $\theta = -38^\circ$ <sup>3</sup> the ratio of photocouplings of the  $N(1535)S_{11}$  resonance increases from  $-0.69$  to  $-0.81$ <sup>3</sup>.

In Table 1 we compare these values with some model calculations. Whereas for the ratio of the photocouplings of the  $N(1535)S_{11}$  resonance there is relatively little variation between the various theoretical results, for the ratio of the proton photocouplings of the  $N(1520)D_{13}$  resonance there is a large spread in values. We also note the large discrepancy between the photocouplings obtained from pion-photoproduction<sup>11</sup> and the new values determined from eta-photoproduction<sup>6,7</sup>.

## 5.2 Transition form factors

There is currently much interest in the  $Q^2$  dependence of the transition form factors. In Fig. 1 we show the  $N(1535)S_{11}$  proton helicity amplitude  $A_{1/2}^p$  for which there exist interesting new data<sup>8</sup> (diamonds). The other points are obtained from a reanalysis of old(er) data, but now using the same values of the resonance parameters for all cases. The solid and dashed curves represent the results of the collective model which were obtained by introducing a mixing angle of  $\theta = -38^\circ$  and  $0^\circ$ , respectively. Just as for the photocouplings, the introduction of the mixing angle improves the agreement with the data.

## 5.3 Tensor analyzing power

Recently, we studied the tensor analyzing power  $T_{02}$  of the process  $d+p \rightarrow d+X$  for forward deuteron scattering in the framework of  $\omega$  exchange using the collective model for the nucleon resonances<sup>9</sup>. Polarization variables, such as the tensor analyzing power, are sensitive to the ratio  $r = \sigma_L/\sigma_T$  of the cross sections for the absorption of virtual isoscalar photons with longitudinal and transversal polarizations by nucleons<sup>24</sup>. Since the lowlying negative parity resonances  $N(1520)D_{13}$ ,  $N(1535)S_{11}$  and  $N(1650)S_{11}$  have only isovector longitudinal form factors, the tensor analyzing power is especially sensitive to the

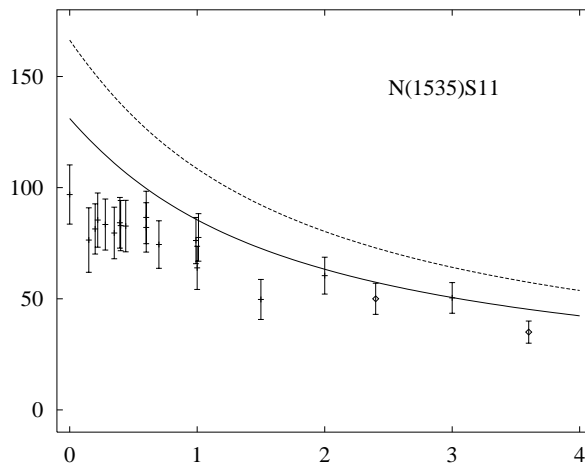


Figure 1:  $N(1535)S_{11}$  proton helicity amplitude in  $10^{-3} \text{ GeV}^{-1/2}$  as a function of  $Q^2$  in  $(\text{GeV}/c)^2$ . A factor of  $+i$  is suppressed. The data are taken from a compilation in<sup>8</sup>. The solid and dashed curves correspond to mixing angles  $\theta = -38^\circ$  and  $0^\circ$ , respectively.

nonvanishing isoscalar longitudinal form factor of the  $N(1440)P_{11}$  Roper resonance. Without the excitation of the Roper resonance  $r = 0$ , and the value of  $T_{20}$  becomes a constant:  $T_{20} = -1/\sqrt{8}$ , (dashed-dotted line in Fig. 2) in disagreement with the existing data (open symbols in Fig 2). The dotted line includes only the Roper resonance, whereas the solid line includes the negative parity resonances  $N(1520)D_{13}$ ,  $N(1535)S_{11}$  and  $N(1650)S_{11}$  as well. Fig. 2 shows that a good description of the data is obtained when  $r$  is calculated using collective form factors.

## 6 Strong couplings

Next we consider strong decays of baryons by the emission of a pseudoscalar meson

$$B \rightarrow B' + M . \quad (10)$$

Here we use an elementary emission model in which the meson is emitted from a single constituent<sup>18,25</sup>

$$\mathcal{H}_s = \frac{1}{(2\pi)^{3/2}(2k_0)^{1/2}} 6X_3^M$$

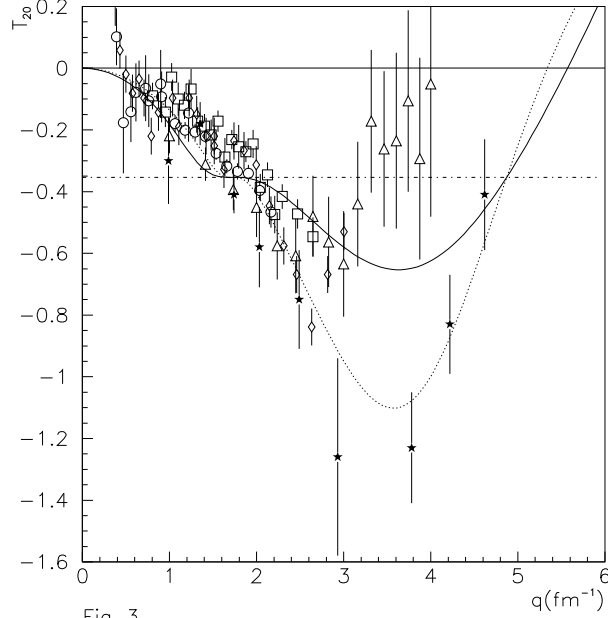


Fig. 3

Figure 2: Tensor analyzing power  $T_{20}$  for  $e^- + d \rightarrow e^- + d$  elastic scattering (filled stars) and  $d + p \rightarrow d + X$  at incident momenta of 3.75 GeV/c (open diamonds), 5.5 GeV/c (open circles), 4.5 GeV/c (open squares), 9 GeV/c (open triangles). The data are taken from a compilation in <sup>9</sup>. The curves represent the results in an  $\omega$ -exchange model with  $r = 0$  (dashed-dotted line), and  $r$  calculated using collective form factors for the Roper resonance only (dotted line) and for the Roper, N(1520), N(1535) and N(1650) resonances (solid line).

$$\left[ (gk - \frac{1}{6}hk)s_{3,z}\hat{U} - hs_{3,z}\hat{T}_z - \frac{1}{2}h(s_{3,+}\hat{T}_- + s_{3,-}\hat{T}_+) \right] . \quad (11)$$

The flavor operator  $X_3^M$  corresponds to the emission of an elementary meson by the third constituent. For the pseudoscalar  $\eta$  mesons we introduce a mixing angle  $\theta_P = -23^\circ$  between the octet and singlet mesons<sup>11,26</sup>. The strong decay widths are calculated in the rest frame of the decaying resonance<sup>27</sup>.

We consider strong decays of nonstrange baryons into the  $\pi$  and  $\eta$  channels<sup>25</sup>. The decay widths depend on the parameters  $g$  and  $h$  in Eq. (11) and on the scale parameter  $a$  of Eq. (6). These parameters were determined from a least



Table 2:  $N^* \rightarrow N + \pi$  and  $N^* \rightarrow N + \eta$  decay widths of 3\* and 4\* resonances in MeV. n.o. stands for not observed. The experimental values are taken from<sup>11</sup>.

| Baryon  | $\Gamma(N^* \rightarrow N\pi)$ |              | $\Gamma(N^* \rightarrow N\eta)$ |             |
|---|--------------------------------|--------------|---------------------------------|-------------|
|   | th                             | exp          | th                              | exp         |
| N(1520) $D_{13}$  | 115                            | $67 \pm 9$   | 1                               | n.o.        |
| N(1535) $S_{11}$  | 85                             | $79 \pm 38$  | 0                               | $74 \pm 39$ |
|   | $20^a$                         |              | $0^a$                           |             |
| N(1650) $S_{11}$  | 35                             | $130 \pm 27$ | 8                               | $11 \pm 6$  |
|   | $142^a$                        |              | $0^a$                           |             |
| N(1675) $D_{15}$  | 31                             | $72 \pm 12$  | 17                              | n.o.        |
| N(1680) $F_{15}$  | 41                             | $84 \pm 9$   | 1                               | n.o.        |
| N(1700) $D_{13}$  | 5                              | $10 \pm 7$   | 4                               | n.o.        |
| N(1720) $P_{13}$  | 31                             | $22 \pm 11$  | 0                               | n.o.        |
| N(2190) $G_{17}$  | 34                             | $67 \pm 27$  | 11                              | n.o.        |
| N(2220) $H_{19}$  | 15                             | $65 \pm 28$  | 1                               | n.o.        |
| N(2250) $G_{19}$  | 7                              | $38 \pm 21$  | 9                               | n.o.        |
| N(2600) $I_{1,11}$  | 9                              | $49 \pm 20$  | 3                               | n.o.        |
| <sup>a</sup> Calculated with a mixing angle of $\theta = -38^\circ$ |                                |              |                                 |             |

square fit to the  $N\pi$  partial widths (which are relatively well known) with the exclusion of the N(1535) $S_{11}$  and N(1650) $S_{11}$  resonances. For the latter, the situation is not clear due to possible mixing of different  $S_{11}$  states or even the existence of a third  $S_{11}$  resonance<sup>28</sup>. As a result we find  $g = 1.164 \text{ GeV}^{-1}$  and  $h = -0.094 \text{ GeV}^{-1}$ . The relative sign is consistent with a previous analysis of the strong decay of mesons<sup>26</sup> and with a derivation from the axial-vector coupling (see<sup>27</sup>). The value of the scale parameter  $a = 0.232 \text{ fm}$  is found to be equal to the value extracted in the calculation of the electromagnetic couplings<sup>14</sup>. The coefficients  $g$ ,  $h$  and  $a$  are kept equal for *all* resonances and *all* decay channels.

In Table 2 we show the decay widths of the nucleon resonances into the  $N\pi$  and  $N\eta$  channels. The  $N\pi$  decay widths are found to be in fair agreement with experiment. For the  $N\eta$  channel our calculation gives systematically small values, whereas the PDG compilation<sup>11</sup> lists a large  $\eta$  width for the N(1535) $S_{11}$  resonance. We emphasize that the coefficients  $g$  and  $h$  in the

transition operator were determined from the  $N + \pi$  decays, so that the  $\eta$  decays are calculated without introducing any further parameters. The  $\eta$  decays are suppressed relative to the  $\pi$  decays because of phase space factors.

The results of our analysis suggest that the large  $\eta$  width that is usually attributed to the  $N(1535)S_{11}$  resonance is not due to a conventional  $q^3$  state. One possible explanation is the presence of another state in the same mass region, *e.g.* a quasi-bound meson-baryon  $S$  wave resonance just below or above threshold, for example  $N\eta$ ,  $K\Sigma$  or  $K\Lambda$  <sup>29</sup>. Another possibility is an exotic configuration of four quarks and one antiquark ( $q^4\bar{q}$ ).

## 7 Summary and conclusions

In this contribution we have analyzed the masses, electromagnetic and strong couplings of baryon resonances in a collective model of the nucleon, in which the baryons are interpreted as rotations and vibrations of a symmetric top with a prescribed distribution of the charge and magnetization.

A study of the mass spectrum showed that the third of four  $P_{11}$  resonances in the recent Zagreb analysis<sup>12</sup> can be assigned as the lowest missing resonance that arises in the collective model. Far more sensitive tests of baryon models are provided by the electromagnetic and strong couplings. As an example we studied the recently determined ratios of photocouplings of the  $N(1535)$  and  $N(1520)$  resonances. Especially for the  $N(1520)$  resonance there is a large variation in the model predictions. Both ratios are found to be in good agreement with the predicted values of the collective model. The same conclusion holds for the transition form factor of the  $N(1535)$  resonance. Another example is the tensor analyzing power in deuteron scattering which is very sensitive to the isoscalar longitudinal form factor of the Roper resonance. It was shown that the use of collective form factors gives the correct dependence of  $T_{20}$  on the momentum transfer.

Whereas the helicity amplitudes for photo- and electroproduction of the  $N(1535)$  resonance are described well, there is a large discrepancy for the strong couplings. Our analysis of the strong decay widths shows that while the  $\pi$  decays follow the expected pattern, the decays into  $\eta$  exhibit some unusual features. Our calculations do not show any indication for a large  $\eta$  width, as is observed for the  $N(1535)$  resonance. This seems to indicate the presence of configurations other than  $q^3$  in the same mass region.

The nature of the  $N(1535)$  resonance has been addressed recently by many authors (see *e.g.* <sup>7,29,30,31</sup>). For example, in the chiral meson-baryon Lagrangian framework of<sup>29</sup> it was found that the  $N(1535)$  ‘could well be a strong background instead of a clean resonance’; in<sup>30</sup> it was shown that speed plots

show no structure in the  $S_{11}$  partial wave at 1535 MeV, but only the strong  $N\eta$  cusp and a resonance at 1650 MeV.

In conclusion, we have shown that the collective model of baryons provides a good overall description of the available data. The nature of the N(1535) resonance remains an open and intriguing question.

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